THIRDS TEACHER NOTES

$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots=\frac{1}{3}$$

Each term is made by multiplying the previous term by $\frac{1}{4}$

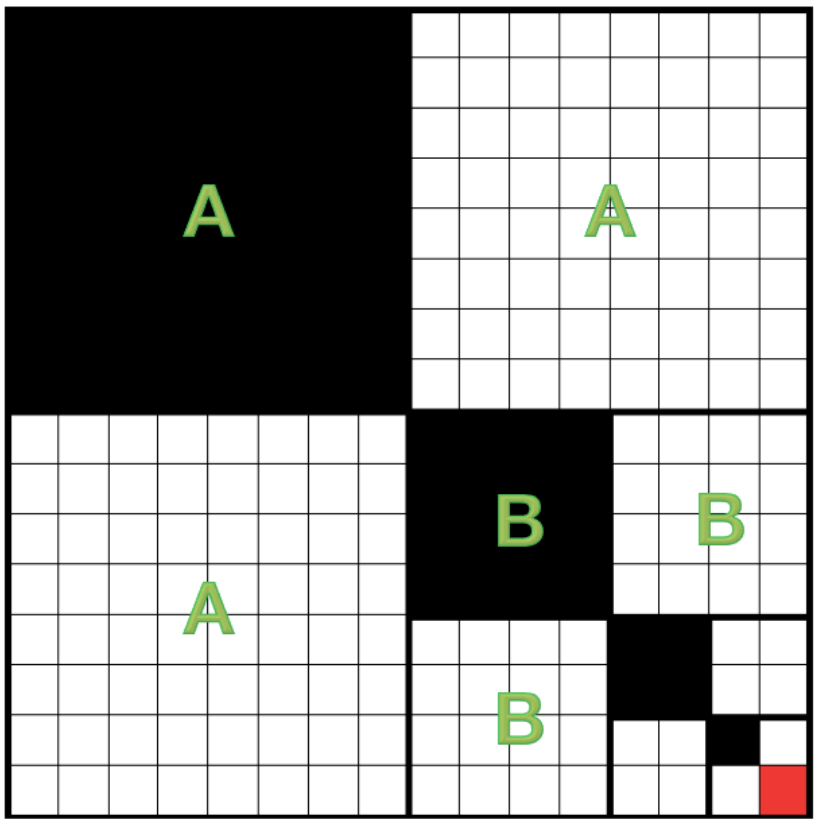
The next fraction in the sum is $\frac{1}{256}$

Shading the Square

The shaded areas below represent each term of the sum, up to the fourth term.

We can see that if we ignore the bit that is left over and yet to be divided (in red), there are three of each 'size of square' (labelled by the same letter).

As one out of every three squares of the same size has been shaded, we can conclude that for an infinite number of terms (when there is no red left over area) the shaded area would be equal to $\frac{1}{3}$



You may wish to colour the first quarter for students and tell them to continue shading in the same way, so that their shading easily lends itself to the visual 'proof' above.

Otherwise, they could find the total fraction they have shaded by counting squares, or by adding the fractions using the grid as a visual aid.

If students do not sub-divide any squares they will have added the first 4 terms which gives a total shaded area of: $\frac{85}{256}$ whilst $\frac{85}{255} = \frac{1}{3}$

To add successive terms students can sub-divide the squares of the grid, continue shading and re-calculate the fraction they have shaded. Each time students can compare their new fraction to $\frac{1}{3}$.

Other methods

In part d) students could find the value of the sum on a spreadsheet by writing a formula to generate successive terms and using the sum function.

A-level students may be able to prove that the sum gives a $\frac{1}{3}$ by using theory about infinite geometric series!