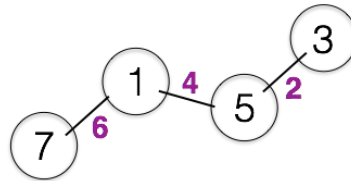




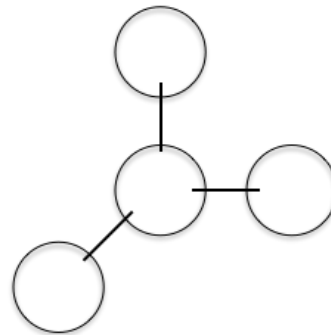
GRACEFUL CREATURES

This snake is a 'graceful' creature.

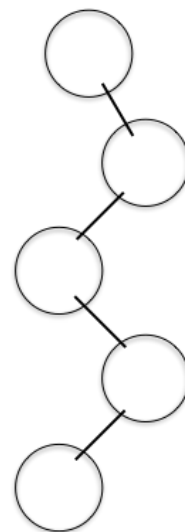
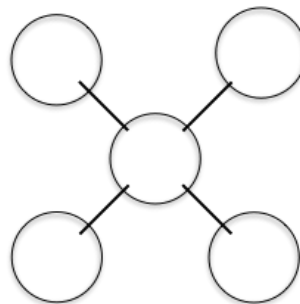
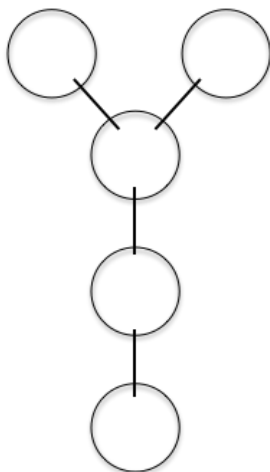


We call it graceful because we can find a way to write **evenly-spaced numbers** (for example 1,3,5,7) in its nodes (circles), so that the **differences** between the pairs of adjacent numbers are **all different**.

1) Show that the creature below is graceful by writing the numbers 1,3,5,7 so that the **differences** between the pairs of adjacent numbers are **all different**.



2) There are only three possible five-node creatures (drawn below). **Show** that all the five-node **creatures are graceful**. Use any five **evenly-spaced** numbers.

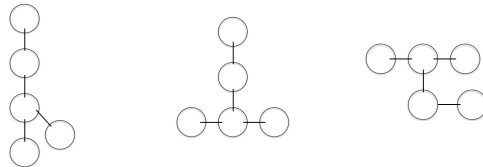




3) In the space below, show that all **six-node creatures** are graceful. You'll need to start by drawing out all the different six-node creatures.

Hints:

- In graph theory, we call these creatures 'trees'. In trees, all nodes are connected, and there are no closed loops.
- If the branches of a tree are bent, or the tree is rotated or reflected, this doesn't change the tree. It will still be the same tree. For example, all three below are the same tree. Can you see why?



4) Mathematicians have proved that all creatures/trees with 35 or fewer nodes are graceful, but have not yet proved this for higher numbers of nodes.

However, we do know that all sizes of 'spider', where one central node is connected to all other nodes, as in the example on the right, are graceful. Explain why this must be the case.

