Fractals Activities: Answer Key

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1) 70.2% of the area is left.

2) Assume the side of big square is 1 unit; then

$$\text{perimeter of fourth stage} = 4 + \frac{4}{3} + \frac{32}{9} + \frac{256}{27} + \frac{2048}{81}$$

Or using finite geometric series formula: $\text{perimeter} = 4 + \dfrac{4\left(1 - \frac{8^4}{3}\right)}{\left(1 - \frac{8}{3}\right)}$

BONUS: $4 + \dfrac{4\left(1 - \frac{8^6}{3}\right)}{\left(1 - \frac{8}{3}\right)} = 290.875$

3) For the total length of the set to be less than one fifth, at least four stages are needed - n must be greater than 3.939.

4) The broccoli is self-similar like a fractal, but true fractals continue for an infinite number of stages.

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1) 9 triangles

2) There are 27 triangles in the third stage, which is 18 more.

3) The number of triangles at each stage is 3^n, where n is the stage number. The pattern goes:

Stage number	0	1	2	3	4	5	6	7
Number of triangles	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$	$3^6 = 729$	$3^7 = 2{,}187$

4) $\frac{3}{4}$, $\frac{9}{16}$; BONUS: $\frac{37}{64}$

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1) The second stage would be 16cm and the third would be $\frac{64}{3}$ cm, which is 21.$\dot{3}$cm.

The length at stage n is the sum of n terms of this sequence:

$$= 12 + (\frac{12}{3} \times 4^0) + (\frac{12}{3^2} \times 4^1) + (\frac{12}{3^3} \times 4^2) \ldots$$

$$= 12 + \sum_{1}^{n} \frac{12}{3^n} \times 4^{n-1}$$

2) 4 triangles are added; 16 triangles are added; after the n^{th} stage the number of triangles added is 4^{n-1}

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1) 20 cubes and 400 cubes; each cube would measure 1cm.

BONUS: The percentage of cube removed is $\frac{7 \times 20}{27 \times 27} = 19.2\%$

2) 16 tetrahedra; 64 tetrahedra; the number of small tetrahedrons in stage n is 4^{n-1}

3) The length of one side of the whole tetrahedron cannot be bigger than 63cm, otherwise the hole would be too big.

If each small tetrahedron measured 10cm, you couldn't have a Level 3 tetrahedron with 8 along each edge, as this would be too big, so the largest would be a Level 2 consisting of 16 tetrahedra.