



TEACHER NOTES: GRACEFUL CREATURES

Graceful Creatures Worksheet - Background

These activities are aimed at Key Stage 3, but could be used with younger or older students. The activities are inspired by the Numberphile video 'The Graceful Tree Problem', here: <http://bit.ly/2V6j54z>

What we have named 'creatures' are called 'trees' in graph theory. The Graceful Tree Conjecture referenced in the video states that all trees (with connected nodes and no closed loops) are graceful, regardless of how many nodes they have.

Graceful Creatures Worksheet - additional activity/discussion

The video, and our first example, use consecutive odd numbers in the nodes. But any evenly-spaced numbers can be used.

An interesting side activity/discussion is to ask the students to prove that if a creature/tree is graceful for one set of evenly-spaced numbers, then it is **graceful for all sets of evenly-spaced numbers**.

Perhaps, if the first creature is shown to be graceful with **1,3,5,7**, then the class could be asked to do the same, but this time with:

- a) 2,4,6,8
- b) 1,2,3,4
- c) 10,20,30,40

Did their method change for the different sets of numbers? Why not?

Students might comment that the same method works because a) the differences are all the same, b)/c) as all the differences have been multiplied by the same thing, they are still all different from each other.

Can they **prove algebraically** that the creature will be **graceful for all sets of evenly-spaced numbers**?

Students could fill in the nodes with:

n for the lowest number in the set,

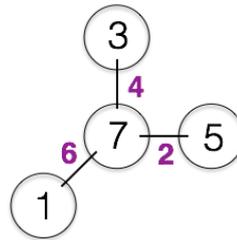
n + d for the second lowest

and so on, and then show algebraically that all the differences are different.



Graceful Creatures Worksheet - Task 1

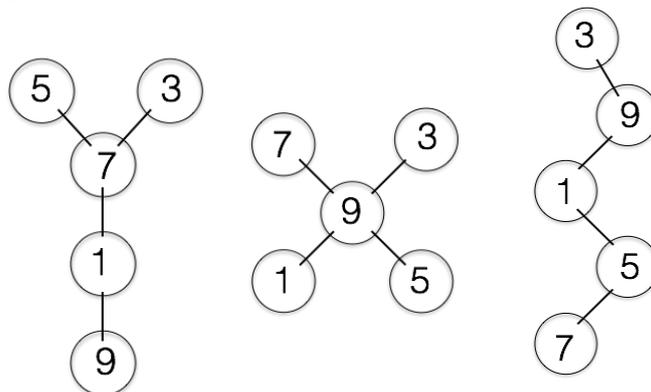
A solution using 1,3,5 and 7:



Graceful Creatures Worksheet - Task 2

In task 2) and 3) the teacher can give students direction as to which numbers (or algebra) to fill the nodes with, or the students can choose their own evenly-spaced numbers.

Solutions using 1,3,5,7 and 9 are below:



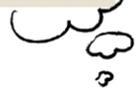
Teachers could discuss with students their **strategies** for filling in the nodes. One key thing to spot is that all the possible differences need to be made, and the largest difference can only be created by putting the largest and smallest numbers adjacent to each other.

Graceful Creatures Worksheet - Task 3

In this creature-drawing task, teachers should encourage the students to develop a **systematic approach**, so they can be sure they have found them all. Their diagrams don't have to look like creatures! (But equally, this could be fun).

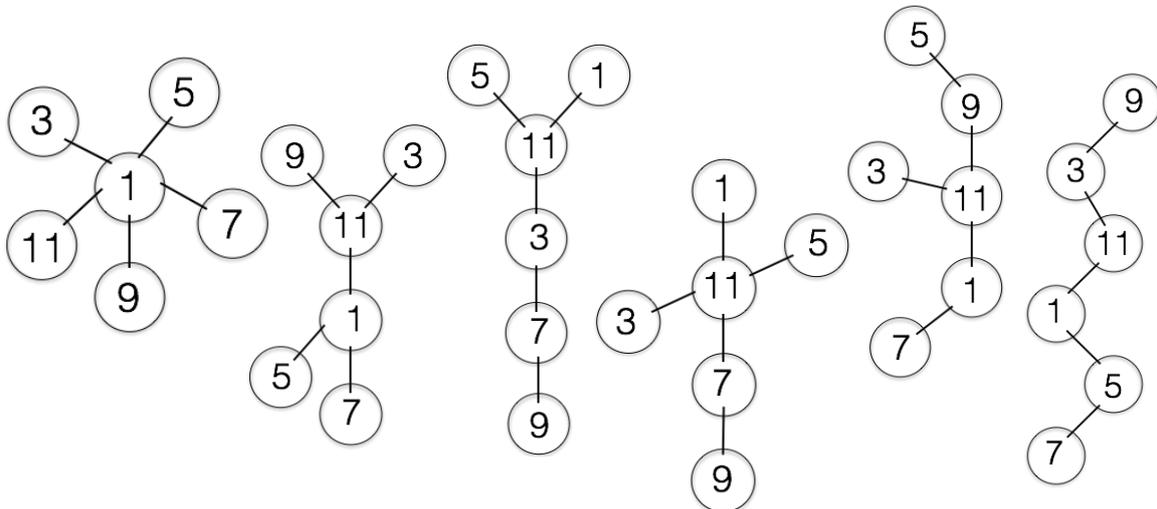
It's very easy to accidentally create creatures/trees that look different, but are in fact the same.

A class discussion could involve students being challenged to spot (from some students' diagrams) creatures that are actually the same, and explain how they know (for example, 'it's been rotated by 180 degrees').



There are only 6 different six-node creatures/trees. If you wish students to find fill the nodes in, but they have not drawn the creatures themselves, the 'Difference Trees' worksheet contains blank copies of these diagrams.

See the creatures and their graceful solutions below.



The Numberphile video shows all the possible seven-node creatures/trees, should you wish to challenge your students to go one node further!

Graceful Creatures Worksheet - Task 4

For the 'spider' there is a strategy that will make it graceful every time. Namely, putting the lowest or highest number in the centre. All the other numbers are different from each other which means they will each be a different distance away from the centre number. This strategy ensures all the possible differences are being made.

Difference Trees Worksheet

This worksheet mostly contains **blank five-node and six-node trees** so teachers can provide the challenge of filling in nodes such that differences are different, without introducing the language of a tree being 'graceful'.

The instructions tell students to use consecutive odd numbers, but students could be instructed otherwise by the teacher if this task also serves as practice of using particular types of numbers.